A STUDY OF THE EFFECT OF FLUID CONTENTS AND INSULATION ON THE VIBRATORY BEHAVIOR OF PIPE

WILLIAM E. McGARRAH

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by

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Lieutenant Commander, United States Navy

Submitted in partial fulfillment of

the requirements for the degree of

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IN

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ABSTRACT

The development of high speed computing devices and the perfection of matrix methods has made it possible to perform quantitative analysis of the vibration characteristics of complex piping systems. Such analyses require the use of accurate input data, such as distributed mass and flexural rigidity.

In this study experiment was compared with theory in a free-free beam configuration in order to assess the confidence that can be attached to values of flexural rigidity and mass distribution calculated from tabulated and actual dimensional and weight data. In addition, data was obtained to assess the damping characteristics.

ACKNOWLEDGEMENTS

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LIST OF SYMBOLS AND ABBREVIATIONS

$A_0, A_1, --- A_n$	Vibration amplitudes, hundredths of an inch
C_1, C_2, C_3, C_4	Arbitrary constants in differential equation
D_o	Outside diameter, in
D_i	Inside diameter, in
E	Modulus of elasticity, psi
I	Moment of inertia, in^4
EI	Flexural rigidity, lb in^2
K	Constant (= 2P/wb)
L	Effective length of pipe, in or ft
M	Bending moment, lb-in
P	Concentrated weight at tips of beam, lb
Q	Concentrated weight at center of beam, lb
V	Shearing force, lb
a	Constant ($a^4 = \omega^2 w/gEI$), in^{-1}
b	One half effective length of pipe (= L/2), in or ft
f	Frequency, cps
g	Acceleration due to gravity, in/sec^2
i	(subscript) - Insulation
ℓ	Average span between supports, in
n	Number of cycles
s	(subscript) - Steel pipe
t	Time (sec) or thickness (in)
u	Variable in differential equation (= ab)

w	Weight per unit length, lb/in or lb/ft
x,y	Variables
eq	(subscript) - Equivalent
δ	Logarithmic decrement
ω	Circular frequency, radians/sec

1. Introduction

The practical difficulties of calculating natural vibration frequencies of complicated piping systems have heretofore been so overwhelming that the attempt was seldom made in practice. Instead, the general theory of vibrations [1,2] [1] was applied in a qualitative way as a guide to design in order to minimize the probability of occurrence of unacceptable vibrations. However, with the development of high speed computing devices and the perfection of matrix-analytical generalizations of Holzer-Prohl-Mykelstadt procedures by E.C. Pestel [3] and his associates, it has now become possible actually to perform quantitative analysis of the vibration characteristics of complex piping systems. Since the theory and the automatic computation procedures are both capable of a high order of accuracy, it is reasonable to evaluate the input data, such as distributed mass and flexural rigidity, to a comparable degree of accuracy.

This study was intended primarily to compare theory with experiment in a simple geometrical configuration easily amenable to analytic treatment in order to assess the confidence that could be attached to values of flexural rigidity and mass distribution calculated from tabulated and actual dimensional and weight data. At the same time,

[1] Number in brackets refers to reference in the Bibliography which appears on page 15.

it was decided to obtain data that could be used to assess damping characteristics. Specifically, the investigation was limited to a study of the effect of one sample of insulation and using water as the fluid contents.

Briefly, it is concluded on the basis of the studies reported herein that natural frequency can be predicted quite accurately by analytical means, and that inherent damping in a piping system, even including the effects of insulation and fluid contents, is so slight that it cannot be considered as a reliable and substantial energy sink.

2. Experimental Method

There are two broad techniques available to study the vibratory and damping behavior of materials and structures, the steady state method and the transient method. General laboratory use of either of these methods has the same essential equipment requirements, viz.,

a. Vibration generator capable of producing vibrations over a useful frequency spectrum.

b. Accurate means of determining frequency of vibration.

c. Equipment for display of steady state vibrations and time decay of vibrations.

The steady state method offers the principal advantage of simplicity in computing damping effectiveness. In this method a condition of forced vibrations at a resonant frequency is established and the amplitude of vibration is determined. Then the frequencies on either side of resonance at which the amplitude is one-half the resonance amplitude are determined. The ratio of the resonance band width (difference between half-amplitude frequencies) to the resonant frequency serves as a measure of damping effectiveness and can be related, approximately, to the logarithmic decrement for small damping [4a]. This method offers distinct advantages where measurements of damping effectiveness are to be made over a wide range of fre-

quencies.

In the transient method, vibrations are induced in the specimen and then vibration excitation is suddenly stopped allowing the vibrations to decay. The time decay of vibrations is recorded and from these records the logarithmic decrement, which serves as a measure of damping effectiveness, can be determined.

It would appear at first glance that the transient method offers no advantage over the steady state method. However, such is not the case. If a weight is hung on a beam, energy in the form of strain energy is stored in the beam. If the weight is suddenly released, the beam will vibrate freely with some time decay until all the strain energy that was present is dissipated. At the instant vibration starts, many modes are present. The vibration amplitude of the fundamental mode is by far the largest and for all practical purposes the fundamental is the only frequency remaining after a period of about one second. This effect can be seen in the time decay graphs in Appendix IV. Due to its predominance, it was considered that the fundamental is of the greatest practical importance in most cases. This led to the decision to limit the study to the fundamental. Having narrowed the scope of the investigation, an advantage of the transient method was established. The advantage here lies in the fact that

utilization of the transient method leads to a simple means of exciting vibrations and also to simple instrumentation. Accordingly, the transient method was selected as the method to be used.

In this study a 100 lb weight was hung at the center of the beam span on a short piece of small mild steel wire. The total strain energy thus stored in the beam was on the order of 1 in-lb (see Appendix II). The weight was suddenly released by melting the small wire with a propane torch, thereby setting up free vibrations.

The fundamental frequencies of vibration anticipated in this test were in the 10-30 cps range. This made it possible to use some rather simple instrumentation. In addition to a variety of commercially available vibration pickups, ordinary resistance type strain gages can be used to sense vibrations through the cyclic variation of strains in the material to which the gages are attached. This sensing signal must be amplified and recorded with some sort of dynamic strain measuring and recording equipment. Strain gages were selected as the means of sensing the vibrations. A Brush Strain Analyzer (Model BL-310) with its companion oscillograph was selected to amplify and record the vibrations since the frequency range of this equipment spans the required frequency range.

It was considered desirable that the investigation be

conducted on pipe of sufficient size to render the results reasonably meaningful on an industrial basis. Accordingly, a four inch (nominal) diameter standard pipe was chosen. Anything larger was considered too large to handle with the space and facilities available.

It was essential that the experimental apparatus have a geometry that would make it amenable to a reasonably accurate mathematical analysis. Any beam-like configuration was considered satisfactory. A free-free beam configuration was selected because of the practical manner in which the pipe could be supported at the nodal points with standard pipe hangers. The basic experimental apparatus is somewhat similar in principle to that used by other investigators [5,6]. A full description of the apparatus can be found in Appendix III.

For purposes of mathematical analysis it was necessary to make an assumption establishing end conditions. Approximately two-third of the length of the standard (ASA B16.9) pipe caps used to seal the ends of the pipe had the same cross section as the pipe. This length was added to the length of the pipe and the effective length thus obtained was used in calculations. The weight of the remainder of each cap plus the weld material used in fastening the cap to the pipe was assumed to be a small concentrated weight at each end of the beam. Solution of the differential

equation using this assumption can be found in Appendix I.

After having obtained a solution to the differential equation, it was necessary to evaluate the flexural rigidity to be used in making the desired frequency predictions. The contained fluid might be expected to contribute something to the rigidity since it has finite viscosity and cohesion. The effect of such contribution was considered to be so slight that it was ignored entirely in computations. The relatively small modulus of elasticity of the insulation led to the belief that there would be little contribution to rigidity from this source. Calculation of the equivalent flexural rigidity of the pipe-insulation combination (see Appendix II) confirmed this belief. The equivalent flexural rigidity was found to differ from the flexural rigidity of the pipe alone by about .5%. The small difference in the calculated flexural rigidities implied that reasonably accurate frequency predictions could be obtained by neglecting the stiffness contribution of the insulation as well as that of the water. However, for purposes of comparison, it was decided to include frequencies calculated using both the equivalent flexural rigidity and the flexural rigidity of the pipe alone.

The logarithmic decrement, δ, of the fundamental vibration was chosen as the means of presenting the damping

effectiveness results. By definition $\delta = \frac{1}{n} \ln \frac{A_o}{A_n}$ [1,2]. The decrement is easily determined from oscillograph records of vibration decay. It offers the advantage of being a dimensionless figure which can be applied to any system and it can be readily related to other measures of damping effectiveness [4a]. Details of determining decrements from the experimental record can be found in Appendix IV.

The following test conditions were used during this investigation:

Test Condition	Description
A -----------	Bare pipe suspended at calculated nodal points without insulation and without fluid contents.
B -----------	Bare pipe suspended at calculated nodal points without insulation but filled with water.
C -----------	Pipe suspended at calculated nodal points with insulation applied but without fluid contents.
D -----------	Pipe suspended at calculated nodal points with insulation applied and filled with water.

3. Experimental Results

The results of this investigation are presented in Table I. In addition to the observed experimental results, the predicted fundamental frequencies are presented for comparison purposes.

It is evident from Table I that the observed fundamental frequencies are in good agreement with those predicted by ordinary beam vibration theory. The maximum difference between the observed frequencies and the calculated frequencies based on nominal dimensions is 2.5%. In the case of calculated frequencies based on actual dimensions the maximum difference is less than 1%. The accuracy of the observed frequencies is $\pm$.25cps as read from the oscillograph charts. If we assume that the maximum error exists, the differences above are 3.6% and 2%, respectively, which still shows good agreement with theory. The larger difference found in comparing observed frequencies and the calculated frequencies based on nominal dimensions can be traced to the fact that the tabulated weight data for the insulation is considerably lower (about 20%) than the actual weight. This discrepancy can be attributed, at least in part, to the presence of moisture in the new insulation. Upon heating in service, the discrepancy can be expected to diminish somewhat as the moisture evaporates and the insulation "cures".

TABLE I

EXPERIMENTAL RESULTS

Test Condition	Predicted Frequency cps nominal #	Predicted Frequency cps actual $	Observed Frequency cps	Logarithmic Decrement δ	Static Strain μ in/in
A- Pipe Alone	25.05	25.07	25.2	.0043	27.5
B- Pipe Filled with Water	20.67	20.72	20.8	.0048	26.5
C- Empty Insulated Pipe	21.36 * 21.41 @	20.75 * 20.80 @	20.9	.0068	26.5
D-Insulated Pipe Filled with Water	18.43 * 18.49 @	18.10 * 18.13 @	18.2	.0062	25.5

\# Based on tabulated nominal dimensions
$ Based on actual dimensions
* Employed flexural rigidity of pipe only
@ Employed equivalent flexural rigidity of pipe-insulation combination

In the test of the pipe alone, the measurements of vibration decay were essentially a measurement of the damping capacity or solid friction of the pipe (mild steel). The value of the logarithmic decrement found (δ =.0043) is reasonably consistent with values recorded in the literature for flexural vibrations [4b]. Since one of the objects of this investigation was to determine the effect of the addition of fluid contents and insulation, the effect of possible inaccuracy in the true value of damping effectiveness was not considered important and the value found was considered to be a valid reference for comparative purposes.

The logarithmic decrements found in Test Conditions B, C, and D indicate that water, insulation, and the water-insulation combination make contributions in some degree to the damping effectiveness. It should be pointed out that the water-insulation combination contributed less damping than the insulation alone. Also, in the sample oscillograph record in Appendix IV (Figure 9) some superimposed oscillations can be observed in Test Condition B. An explanation of these effects might be found in a more complete energy analysis of the apparatus, but this was considered to be beyond the scope of this investigation.

Errors in the measured values of damping effectiveness can be attributed to energy losses outside the specimen

itself and errors in determining the logarithmic decrement. Losses outside the specimen were not considered to be important as long as they were held to a consistent minimum because of the comparative nature of the tests. Vibration amplitudes from the oscillograph record were measured to ±.02". With reasonable care in determining logarithmic decrements it is estimated that the values of individual decrements were accurate to within 5%. The significance of this degree of accuracy is diminished when we consider that the spread in logarithmic decrements was greater than 5%. However, this does not in any way weaken the conclusion that the addition of water[2] and insulation result in little practical contribution to damping effectiveness.

By the use of elementary strength of materials we are able to predict (see Appendix II) that the static strain to be expected at the gage locations with a 100 lb load suspended at the center is 27 μin/in. The static strains found experimentally (which are accurate to within $\pm 1 \mu$ in/in) agree essentially with the value predicted.

[2]Lateral accelerations did not exceed g. Contained fluid might provide damping in cases where lateral acceleration is greater than g.

4. Conclusions

From the results of this investigation the following conclusions can be drawn.

Theoretical prediction of the natural frequency of vibration of a piping system involves a knowledge of its geometry, the nature of the constraints, and the evaluation of some combination of unit mass and flexural rigidity. This study avoided the complications which might be associated with a complex geometry and a complicated system of constraints. Attention was focused on the w/EIg parameter which is important in theory. From this study we can conclude that the parameter can be evaluated successfully by actual measurement of the weight and dimensions, and that the fundamental frequency of vibration can be predicted quite accurately. In addition, while the contribution of the insulation to flexural rigidity can be calculated, if desired, it appears from this study that any contributions from fluid contents or insulation are negligible. It is possibly fortuitous in the case of the pipe used in this investigation that tabulated dimensions and weights gave values very close to those actually measured. Agreement in the case of the insulation is not as good but still reasonable. It is not possible to assert on the basis of anything done in this study that such agreement will always be found. How-

ever, it appears reasonable to suggest that, in the absence of more reliable data, good predictions can be made on the basis of nominal dimensions and weights unless there is reason to believe that the actual pipe and insulation in question differ appreciably from nominal.

Moreover, we can conclude that while both fluid contents and insulation covering make a contribution to vibration damping, it is by no means a substantial one. From a practical standpoint we must still rely on other agencies, such as vibration absorbers or the characteristics of the hanger installation, to supply most of the energy absorption.

BIBLIOGRAPHY

1. J.P. Den Hartog, Mechanical Vibrations, 3rd Edition, McGraw-Hill Book Co., 1947.

2. S. Timoshenko, Vibration Problems in Engineering, 2nd Edition, D. Van Nostrand Co., Inc., 1937.

3. E.C. Pestel, Matrix Methods in the Dynamics of Machines and Structures, Lecture Notes, Engineering Extension, UCLA; see also forthcoming McGraw-Hill book by E.C. Pestel and F.A. Leckie.

4. A.L. Kimball, Vibration Problems, ASME Transactions, Vol. 63 (1941).
   a. Part IV - Friction and Damping in Vibrations, p A-37.
   b. Part V - Friction and Damping in Vibrations, p A-135.

5. A. Gemant, The Measurement of Solid Friction of Plastics, Journal of Applied Physics, Vol. 11 (1940), p 647.

6. D.E. Callaway, F.G. Tyzzer, and H.C. Hardy, Resonant Vibrations in a Water-Filled Piping System, Journal of the Acoustical Society of America, Vol. 23 (1951) p 550.

APPENDIX I

SOLUTION OF DIFFERENTIAL EQUATION

The differential equation for a laterally vibrating beam of constant cross section is [1,2]:

$$\frac{\partial^2 y}{\partial t^2} - a^4 \frac{\partial^4 y}{\partial x^4} = 0 \qquad \text{eqn [1]}$$

where

$$a^4 = \frac{\omega^2 w}{g E I} \qquad \text{eqn [1a]}$$

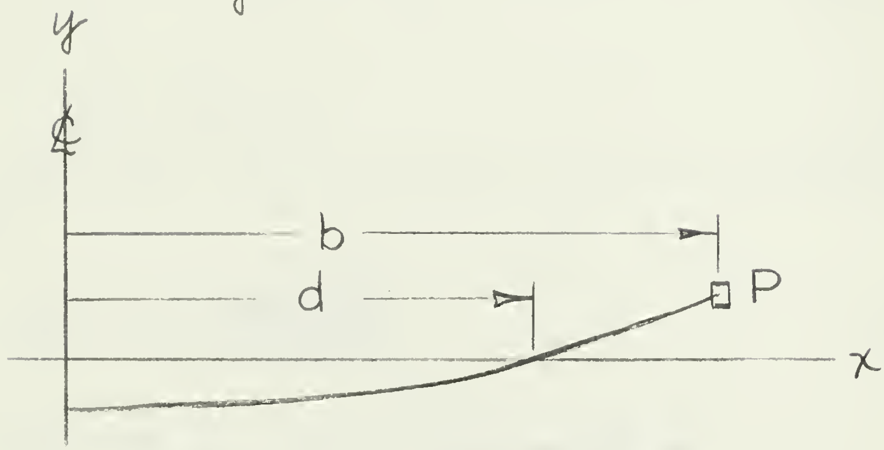


FIGURE 1

Assuming a solution of the form

$$y = \cos \omega t (C_1 \cosh ax + C_2 \cos ax + C_3 \sinh ax + C_4 \sin ax) \qquad \text{eqn [2]}$$

we obtain

$$\ddot{y} = -\omega^2 \cos \omega t (C_1 \cosh ax + C_2 \cos ax + C_3 \sinh ax + C_4 \sin ax) = -\omega^2 y \qquad \text{eqn [2a]}$$

$$y' = a \cos \omega t (C_1 \sinh ax - C_2 \sin ax + C_3 \cosh ax + C_4 \cos ax) \qquad \text{eqn [2b]}$$

$$y'' = a^2 \cos \omega t (C_1 \cosh ax - C_2 \cos ax + C_3 \sinh ax - C_4 \sin ax) = \frac{M}{EI} \qquad \text{eqn [2c]}$$

$$y''' = a^3 \cos \omega t (C_1 \sinh ax + C_2 \sin ax + C_3 \cosh ax - C_4 \cos ax) = \frac{V}{EI} \qquad \text{eqn [2d]}$$

where $\dot{} = \frac{\partial}{\partial t}$ and $' = \frac{\partial}{\partial x}$

For a free-free beam with a concentrated weight, P, at each end the boundary conditions are:

at $x = 0$ $\qquad V = 0$, $y' = 0$ $\qquad$ eqn [3a]

at $x = b$ $\qquad M = 0$, $V = \frac{P}{g}\omega^2 y$ $\qquad$ eqn [3b]

Applying the first set of boundary conditions eqn [3a], we find that

$$C_3 = C_4 = 0 \qquad \text{eqn [4a]}$$

Applying the second set of boundary conditions eqn [3b], we find that

$$C_2 = C_1 \frac{\cosh ab}{\cos ab} \qquad \text{eqn [4b]}$$

and

$$EIa^3[C_1 \sinh ab + C_2 \sin ab]\cos \omega t = -\frac{P}{g}\omega^2[C_1 \cosh ab + C_2 \cos ab]\cos \omega t \qquad \text{eqn [5]}$$

If we use equations [1a] and [4b] and rearrange terms, equation [5] becomes

$$\tanh u + \tan u = -Ku \qquad \text{eqn [6]}$$

where

$$K = \frac{2P}{wb} \qquad \text{eqn [6a]}$$

and

$$u = ab \qquad \text{eqn [6b]}$$

There are an infinite number of solutions to equation [6] and this implies an infinite number of natural frequencies and modes. However, since we are concerned in this investigation with the first mode only, we shall seek only the numerical solution for the lowest value of circular frequency. From this numerical solution the fundamental frequency of vibration for the various test conditions can be obtained.

If we substitute the values of the arbitrary constants, C_1, C_2, C_3, and C_4 into equation [2] we obtain

$$\frac{y}{C_1} = \cosh ax + \frac{\cosh u}{\cos u} \cos ax \qquad \text{eqn [7]}$$

At the nodal points y=0 and equation [7] becomes

$$\frac{\cos u}{\cosh u} = -\frac{\cos ax}{\cosh ax} \qquad \text{eqn [8]}$$

After obtaining a value for u from the numerical solution of equation [6], it is possible to solve for x in equation [8] to determine the nodal points for the various test conditions.

APPENDIX II

MISCELLANEOUS CALCULATIONS

Structural and Geometrical Data

The data listed in Table II were used in all calculations in this section.

Predicted Frequencies

With the aid of five place function tables and a desk calculator, accurate solutions for u in equation [6] of Appendix I were obtained. Only those values of u corresponding to the lowest circular frequency in each test condition were determined. Using equations [6b] and [1a] and the pertinent data listed in Table II the circular frequencies were calculated directly. The frequency in cycles per second was then calculated using the relation

$$\omega = 2\pi f$$

Equations [6], [6b], and [1a] are repeated below.

$$\tanh u + \tan u = -Ku \qquad \text{eqn } [6]$$

$$u = ab \qquad \text{eqn } [6b]$$

$$a^4 = \frac{\omega^2 w}{gEI} \qquad \text{eqn } [1a]$$

These calculations were carried out with two groups of data; first, using the dimensions of the actual pipe used in the investigation, and, second, using the nominal dimensions tabulated in piping catalogs. In addition, two sets of cal-

TABLE II

STRUCTURAL AND GEOMETRICAL DATA

PIPE	ACTUAL	NOMINAL #
OD, in	4.520 $	4.500
ID, in	4.038 $	4.026
t, in	.241 $	.237
w, lb/ft lb/in	11.1 $ .924 $	10.79 .899
I, in^4	7.42 *	7.23
E, psi	30×10^6	30×10^6
WATER		
w, lb/ft lb/in	5.56 * .463 *	5.51 .459
INSULATION		
OD, in	9.50 $	9.50
ID, in	4.50 $	4.50
t, in	2.50 $	2.50
w, lb/ft lb/in	5.50 $ .458 $	4.4 $$ .367 $$
I, in^4	381 *	381 *
E, psi	2650 ##	3300 **

\# Nominal values are those tabulated in catalogs for schedule 40 pipe and Thermobestos insulation.
$ Determined by actual measurement.
* See calculations later in this section.
\## Determined by compression test of sample of insulation used.
$$ Approximate weight furnished by manufacturer.
** Value inferred from manufacturer's data of compressive load to cause specified shortening.

P = 2 lb b = 8.5 ft = 102 in g = 386 in/sec^2

culations were carried out for each group of dimensions in Test Conditions C and D for the purpose of comparison. In the first, the simplification that both the water and the insulation added mass but not rigidity was employed and the flexural rigidity used was that of the pipe alone. In the second set, the contribution of the insulation to rigidity was considered and an equivalent flexural rigidity of the pipe-insulation combination was used. The results of these calculations are tabulated in Table III.

NODAL POINTS

Having calculated u and a for the various test conditions, the location of the points at which to suspend the pipe was determined by solving for x in equation [8] of Appendix I using five place function tables and a desk calculator. Equation [8] is repeated below.

$$\frac{\cos u}{\cosh u} = -\frac{\cos ax}{\cosh ax} \qquad \text{eqn } [8]$$

These calculations were carried out using both the actual and nominal dimensions. The nodal points calculated with actual dimensions were used in this investigation. However, there is little practical difference between those calculated for actual dimensions and those calculated for nominal dimensions. The results are listed in the last column of Table III.

TABLE III

SUMMARY OF FREQUENCY CALCULATIONS

Test Condition	K	u	a in^{-1}	ω rad/sec	f cps	d * in
ACTUAL DIMENSIONS						
A	.04240	2.3182	.02273	157.5	25.07	57.45
B	.02825	2.3332	.02287	130.2	20.72	57.05
C	.02835	2.3330	.02287	130.4# 130.7$	20.75# 20.80$	57.05
D	.02124	2.3408	.02295	113.7# 113.9$	18.10# 18.13$	56.85
NOMINAL DIMENSIONS						
A	.04361	2.3169	.02271	157.4	25.05	57.45
B	.02887	2.3324	.02287	129.9	20.67	57.05
C	.03098	2.3302	.02284	134.2# 134.5$	21.36# 21.41$	57.15
D	.02273	2.3391	.02293	115.9# 116.2$	18.43# 18.49$	56.85

* Distance from center of span to nodal point.
\# Using flexural rigidity of pipe only.
$ Using equivalent flexural rigidity of pipe-insulation combination.

MOMENT OF INERTIA

For a hollow circular cross section

$$I = \frac{\pi}{64}\left[D_o^4 - D_i^4\right]$$

In the case of the actual pipe used in the investigation

$$I = \frac{\pi}{64}\left(4.520^4 - 4.038^4\right) = 7.42 \text{ in}^4$$

In the case of the insulation used in the investigation

$$I = \frac{\pi}{64}\left(9.5^4 - 4.5^4\right) = 381 \text{ in}^4$$

WEIGHT OF WATER PER UNIT LENGTH

In the case of nominal pipe the weight of water per unit length is tabulated along with other nominal data. For the actual pipe used in the tests, this value must be calculated.

The cross-sectional flow area of the actual pipe is

$$\text{Area} = \frac{\pi}{4} D_i^2 = \frac{\pi}{4}(4.038)^2 = 12.82 \text{ in}^2$$

The weight of water per inch of length is found as follows

$$W = (12.82 \text{ in}^2)(1 \text{ in}) \frac{62.4 \text{ lb/ft}^3}{1728 \text{ in}^3/\text{ft}^3} = .463 \text{ lb/in}$$

and

$$W = (.463 \text{ lb/in})(12 \text{ in/ft}) = 5.56 \text{ lb/ft}$$

EQUIVALENT FLEXURAL RIGIDITY

The equivalent flexural rigidity for the pipe-insulation combination can be calculated as follows. For nominal

dimensions:

$$(EI)_s = (30)(10^6)(7.23) = (216,900)(10^3) \ \text{lb in}^2$$

$$(EI)_i = (3300)(381) = (1,257)(10^3) \ \text{lb in}^2$$

$$(EI)_{eq} = (EI)_s + (EI)_i = (216,900 + 1,257)(10^3) \ \text{lb in}^2$$

$$= (218,157)(10^3) \ \text{lb in}^2$$

Similarly, for actual dimensions:

$$(EI)_s = (30)(10^6)(7.42) = (222,600)(10^3) \ \text{lb in}^2$$

$$(EI)_i = (2650)(381) = (1010)(10^3) \ \text{lb in}^2$$

$$(EI)_{eq} = (EI)_s + (EI)_i = (222,600 + 1010)(10^3) \ \text{lb in}^2$$

$$= (223,610)(10^3) \ \text{lb in}^2$$

From these calculations we see that $(EI)_{eq}$ and $(EI)_s$ differ by about .5%. For all practical purposes the insulation does not contribute rigidity but calculations based on both flexural rigidities are included for comparison purposes.

STRAIN ENERGY

In the testing technique used in this investigation, energy in the form of strain energy is stored in the beam and dissipated in the form of free vibrations. From elementary strength of materials we know that the strain energy, U, can be found as follows:

$$U = \int \frac{M^2}{2EI} = \frac{1}{2} Q y$$

where Q is the weight suspended from the center of the beam for exciting vibrations and y is the deflection at the center of the beam.

For the beam in this investigation

$$y = \frac{Q\ell^3}{48EI}$$

where ℓ is the average length of the span between supporting points. Using Q = 100 lb, ℓ = 114 in, and EI = (216.9)(10^6) lb in^2, we can determine that

$$U = \frac{Q^2\ell^3}{96EI} = .715 \text{ in lb}$$

which is the total amount of energy stored in the beam prior to initiation of free vibrations.

STATIC STRAIN

From elementary strength of materials, the strain to be expected at the strain gage location from a 100 lb load suspended at the center of the pipe can be calculated as shown below.

$$\epsilon = \frac{Mc}{EI} = \frac{Q}{2}\left(\frac{\ell}{2} - 5\right)\left(\frac{D_o}{2}\right)\left(\frac{1}{EI}\right)$$

$$= \frac{(50)(52)(2.25)}{(30)(10^6)(7.23)} = 27 \mu in/in$$

APPENDIX III

EXPERIMENTAL APPARATUS AND PROCEDURE

Basic Apparatus

The basic experimental apparatus consisted of a 16'8" length of four inch standard carbon steel pipe (Schedule 40, ASTM A-53, Grade B). The ends were closed with four inch standard ASA carbon steel butt welding caps. One cap was fitted with two one-half inch tapped holes for filling and draining the pipe.

The pipe was suspended at the nodal points for a free-free beam configuration with two ring type pipe hangers. A strip of mild steel 1/16" thick was placed inside the pipe hangers to insure contact around the circumference of the pipe at the suspension points. This strip was used because preliminary tests showed that without contact completely around the circumference an excessive and irregular energy loss resulted.

A third ring type pipe hanger was inverted and attached at the center of the span of pipe. To this hanger was attached a rod and a weldless eye nut to provide a means of suspending the load.

Insulation

The insulation used was Johns-Manville Thermobestos Insulation (2½" thick). It was applied to the full effective length (17'0") of the pipe. The voids around the

hangers were filled with Johns-Manville #450 insulating cement.

In order to render the results more meaningful industrially it was considered desirable to remove any inconsistencies that might arise from faulty or amateur application of the insulation. Accordingly, the actual application of the insulation was performed by an experienced person using techniques commonly employed in the Navy.

The insulation was cut and trimmed for a good fit and strapped into place with standard insulation strapping. The overlapping cheese cloth covering was then smoothed down and secured with wheat paste.

Instrumentation

Resistance type strain gages were used to sense the vibrations. Four Baldwin SR-4 type A-5 strain gages were mounted near the center of the span of pipe as shown in Figure 2.

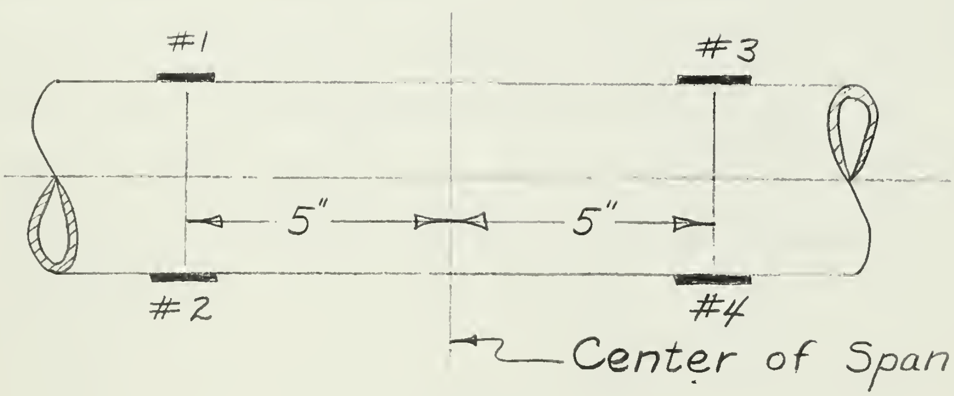


FIGURE 2

For vibration measurements, gages #1 and #2 were

used as a two-gage bridge. Leads from these gages were connected to a Strain Analyzer Model BL-310 (Brush Development Co.). The output was recorded on a companion Brush direct-inking oscillograph.

For static strain measurements a four-gage bridge composed of all four gages was used. Measurements were taken with a Baldwin-Lima-Hamilton Type N Strain Indicator.

Experimental Procedure

Prior to each test the Brush Strain Analyzer was warmed up and balanced in accordance with the manufacturer's instructions. No attempt was made to calibrate the Brush equipment for accurate strain measurements. Instead, maximum gain was used in order to obtain maximum initial amplitude of vibration. The beam was then loaded with a 100 lb weight. A short length of #16 mild steel wire was used to hang the weight on the loading ring. After starting the oscillograph at the desired paper speed, the weight was released by melting the wire with a propane torch. For each set of test conditions four runs were made using a paper speed of 25 mm/sec (Runs #1,2,3,4) and three runs were made using a paper speed of 125 mm/sec (Runs #5,6,7).

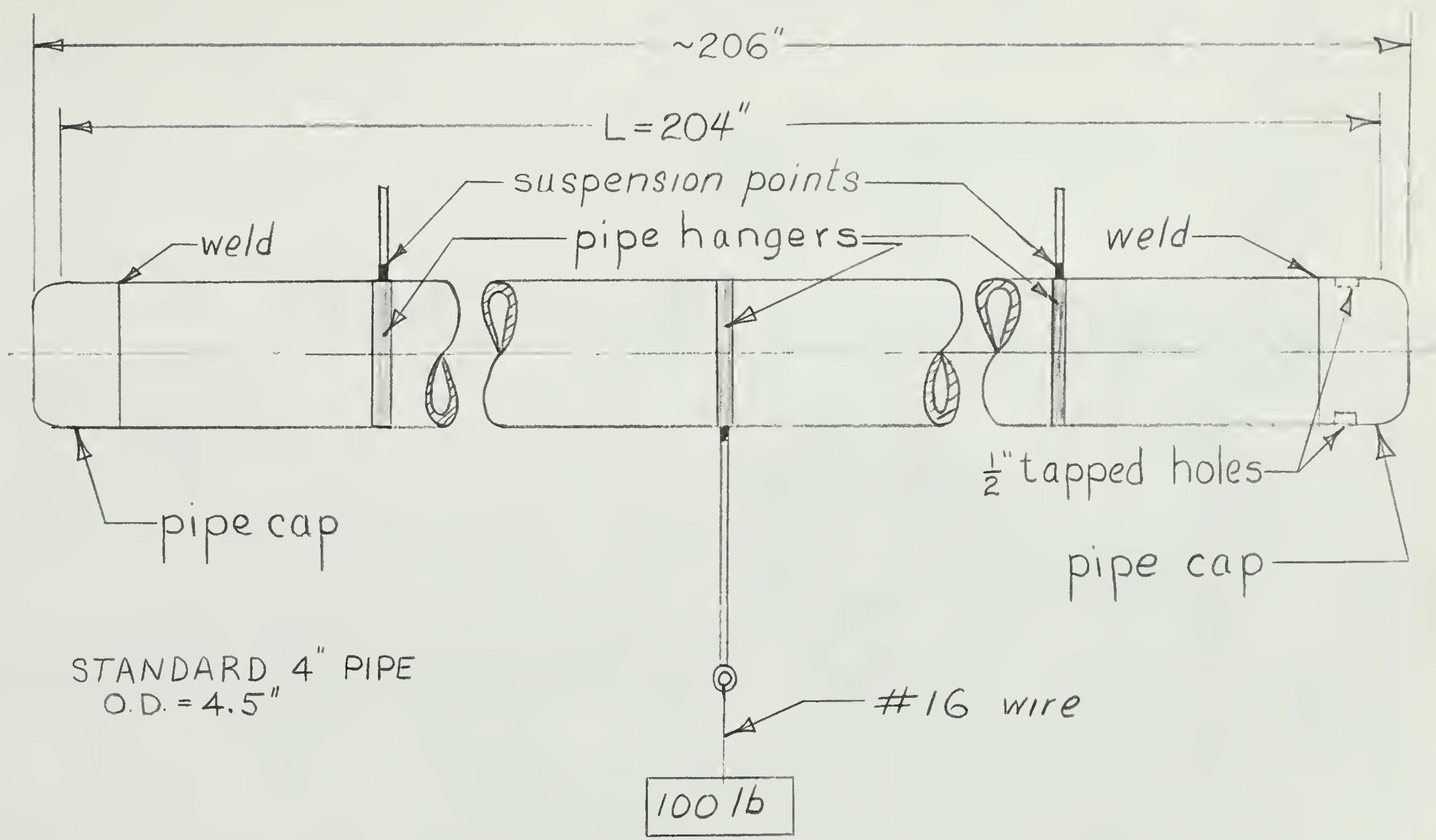

~206"
L=204"
suspension points
pipe hangers
weld
weld
pipe cap
1/2" tapped holes
pipe cap
STANDARD 4" PIPE
O.D. = 4.5"
#16 wire
100 lb

FIGURE 3

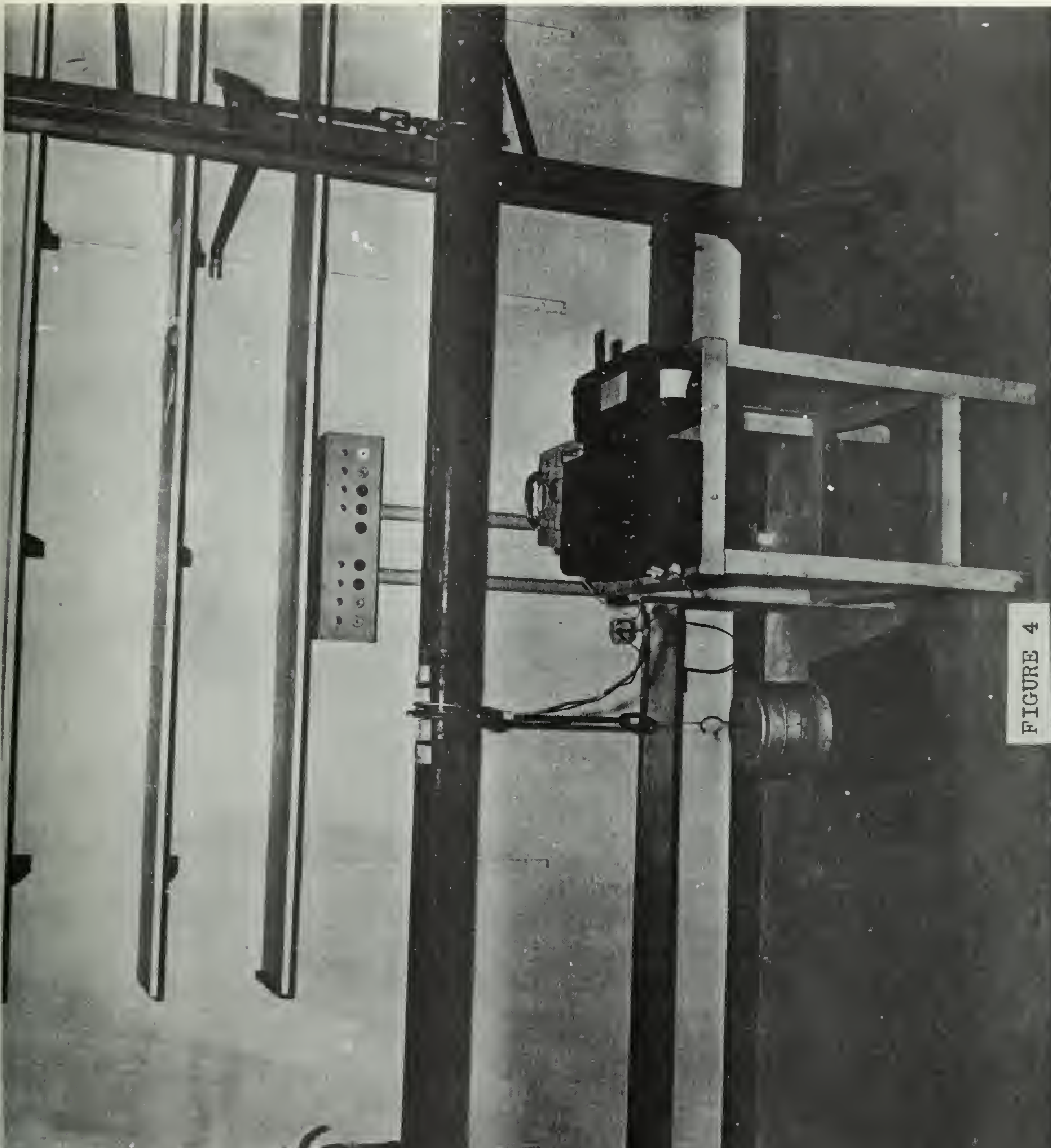


FIGURE 4

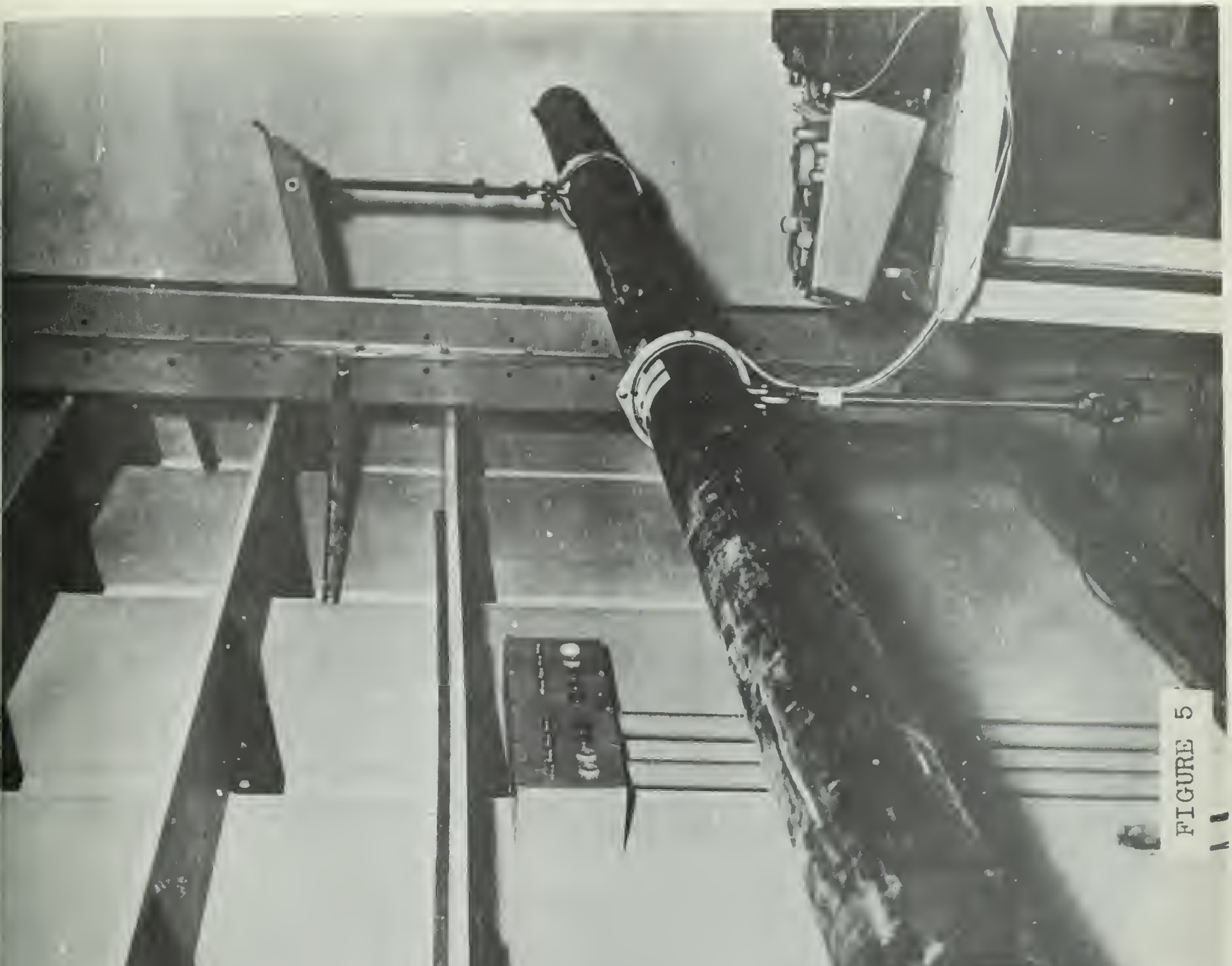


FIGURE 5

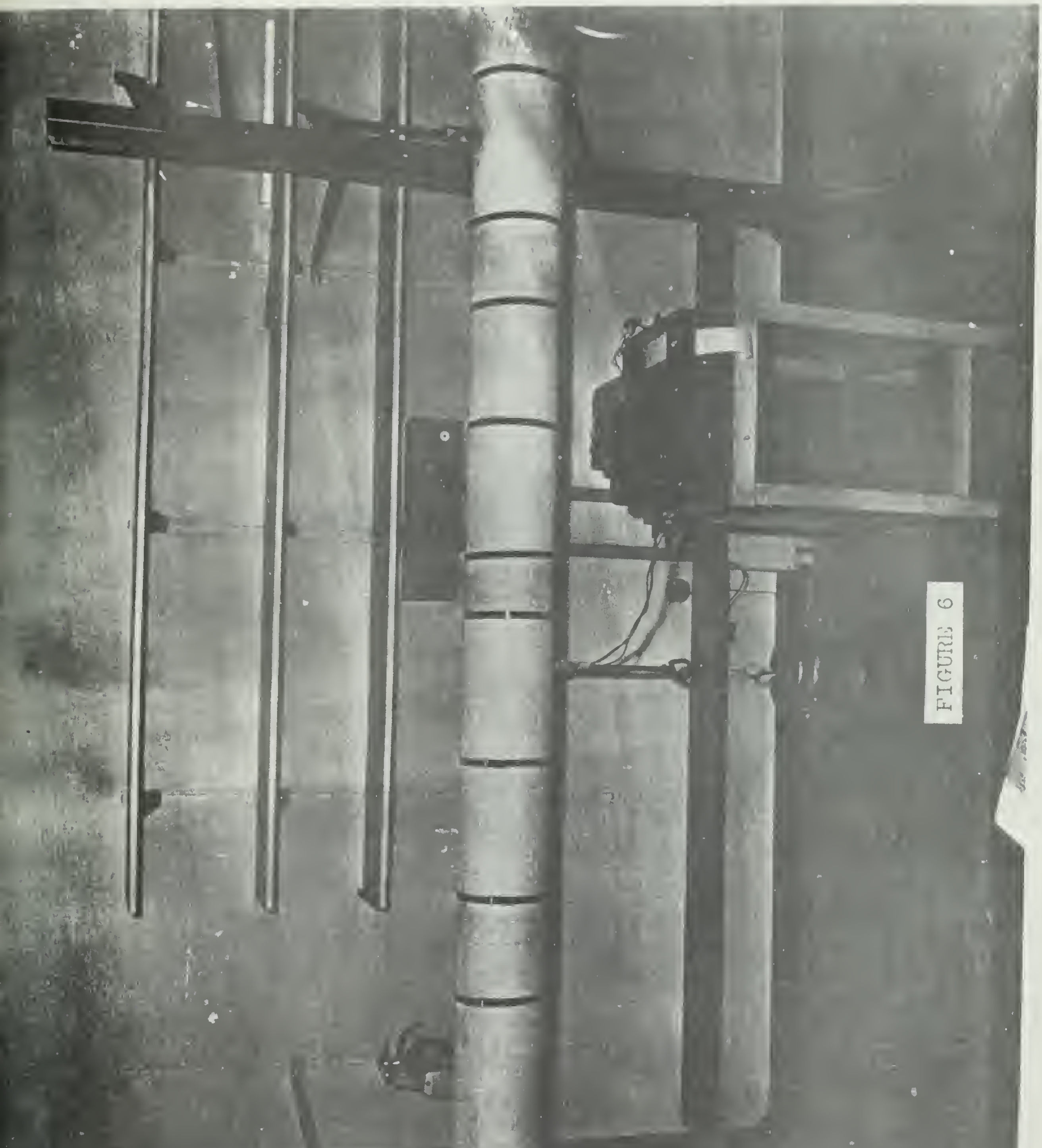


FIGURE 6

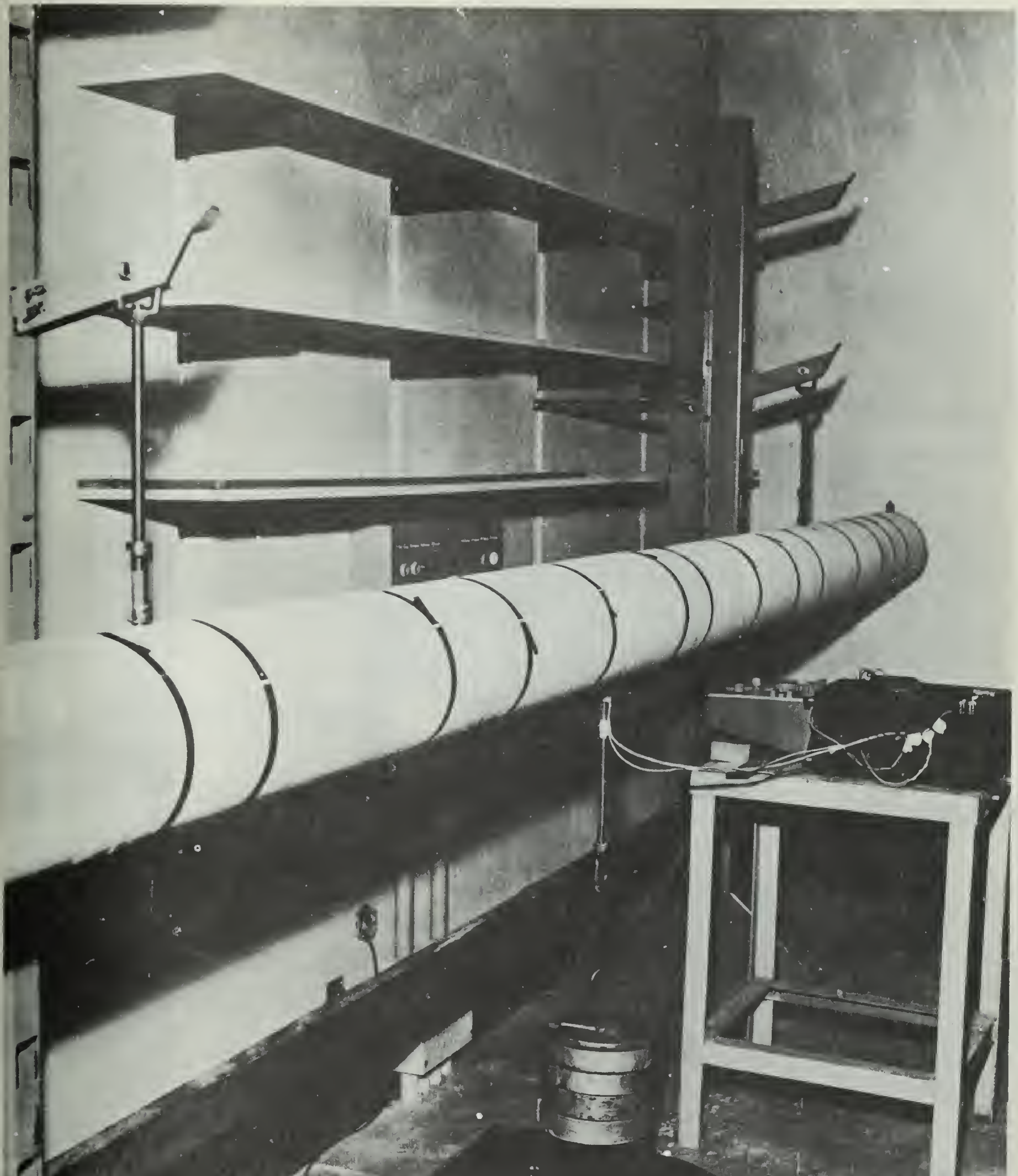


FIGURE 7

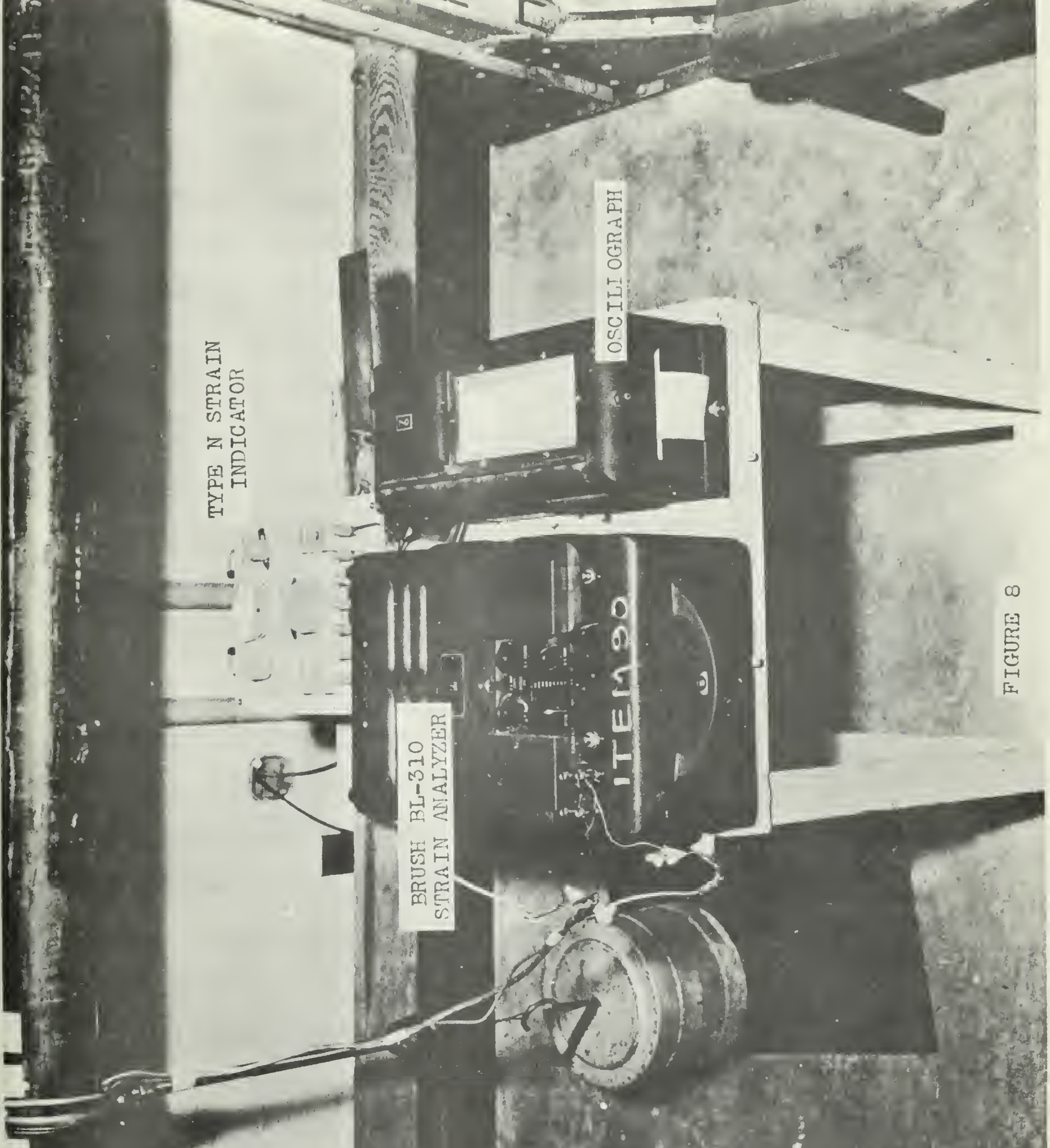

TYPE N STRAIN
INDICATOR
BRUSH BL-310
STRAIN ANALYZER
OSCILLIOGRAPH
ITEM 90

FIGURE 8

APPENDIX IV

REDUCTION OF EXPERIMENTAL DATA

Experimental Data

Table IV is a tabulation of the experimental data.

Experimental Frequencies

The observed fundamental frequency of vibration was determined by counting the number of cycles in a random one second period after at least one second had elapsed on each of the three runs. The frequencies thus obtained actually varied less than 1% and the frequency listed in the table of experimental results is the average of the three frequencies.

Logarithmic Decrement

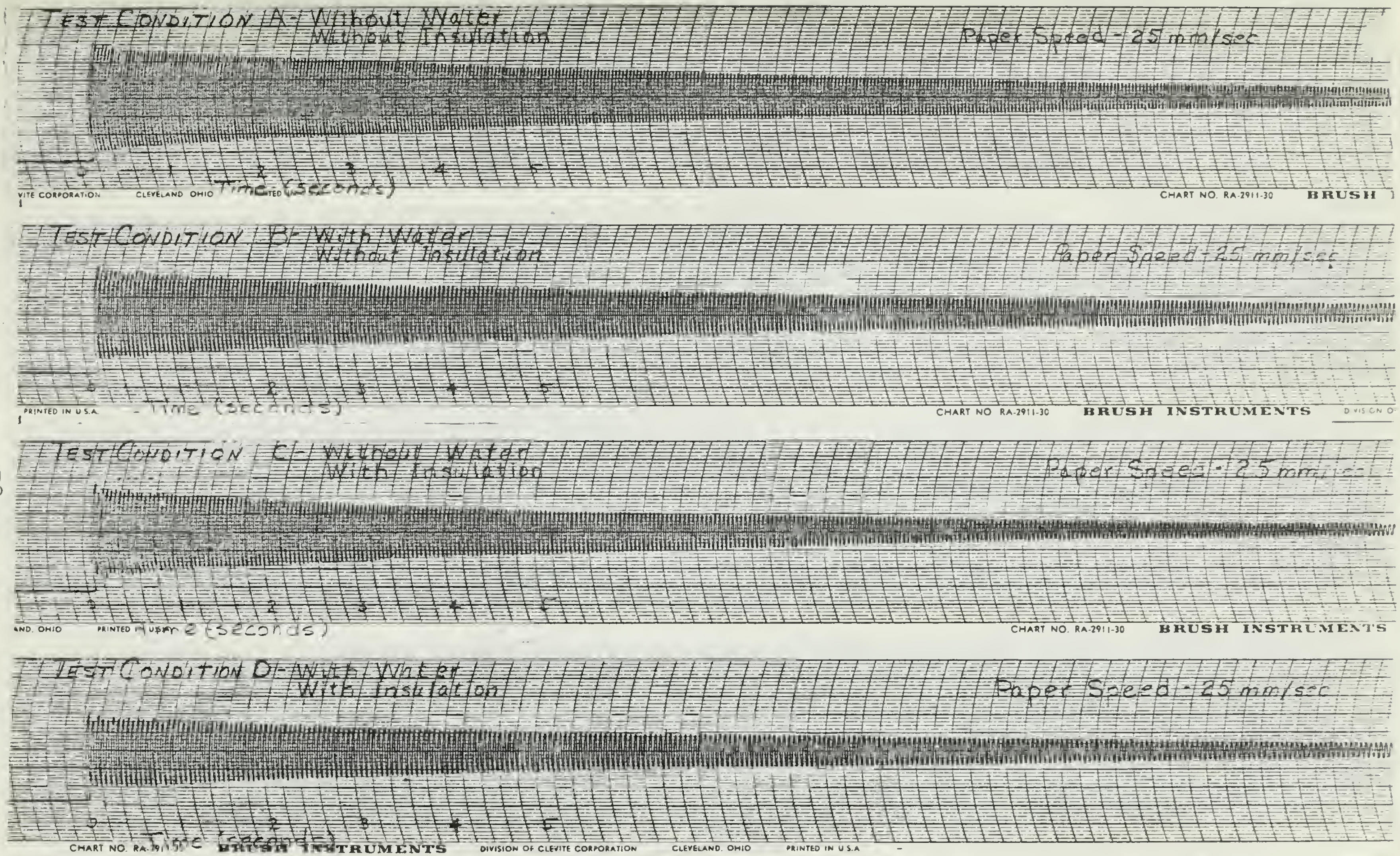
As mentioned earlier, a total of seven runs was made for each set of test conditions. On each run, a zero time for amplitude measurements was selected arbitrarily after at least one second of vibration had elapsed (for reasons previously mentioned). At this point a short portion of the envelope of the vibration decay curve was determined by drawing the best straight line through two or three peaks on each side of zero time. The measured zero amplitude, A_0 , was the distance between the envelope lines on the zero time line. Three more amplitudes (A_n, A_{2n}, and A_{3n}) were measured in the same manner at an elapsed time of n, 2n, and 3n cycles, respectively, from zero time.

From theory we know that the plot of ln A vs n is a straight line. The best value of logarithmic decrement is the slope of the best straight line on the semi-log plot. For each run such a semi-log plot was made and the logarithmic decrement was determined as indicated above using the slope of the best straight line. The average of all seven values for each test condition is the value reported in the results.

TABLE IV

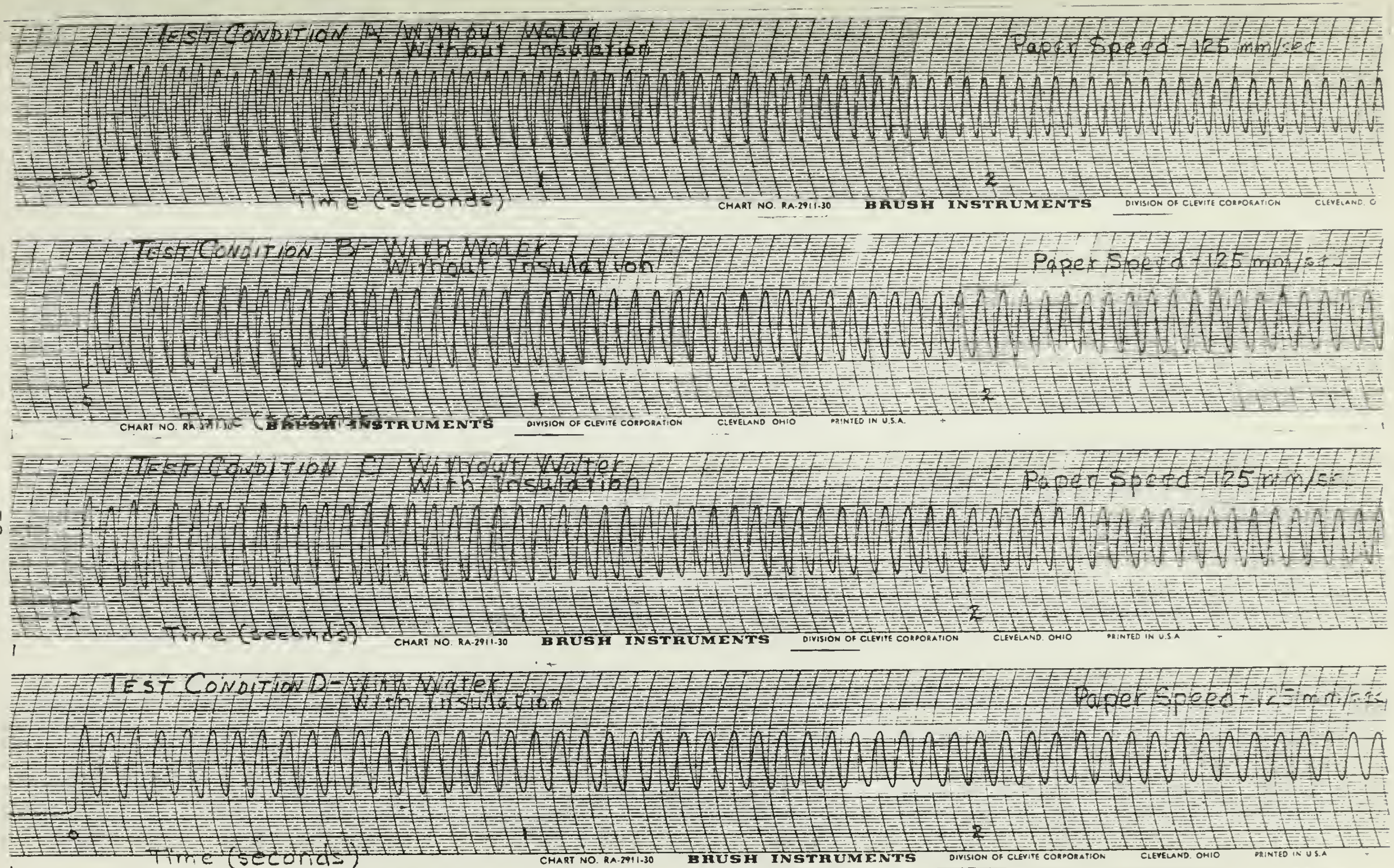
EXPERIMENTAL DATA

Run	f	A_0	A_n	A_{2n}	A_{3n}	n	δ
	cps	hundredths of an inch				cycles	
TEST CONDITION A							
1	--	70.5	55	45	35.5	50	.0045
2	--	69	56	45	35	50	.0043
3	--	65	51.5	43.5	32.5	50	.0043
4	--	65.5	52	43	32	50	.0043
5	25.2	85.5	73	68	62.5	20	.0044
6	25.2	83.5	72.5	67.5	62.5	20	.0040
7	25.2	78	68	64.5	60.5	20	.0040
TEST CONDITION B							
1	--	75	62	50	38	50	.0041
2	--	69	57	43	33	50	.0046
3	--	63	53	38	31.5	50	.0045
4	--	65.5	55	41.5	33	50	.0046
5	20.8	84	74	68	62.5	20	.0050
6	20.8	82	72	63	58.5	20	.0055
7	20.7	82.5	73.5	67	61	20	.0051
TEST CONDITION C							
1	--	62	41.5	30	21	50	.0073
2	--	61.5	43	34	22.5	50	.0067
3	--	66.5	47.5	36	25	50	.0065
4	--	63	46	36	24.5	50	.0063
5	21.0	78.5	66	54.5	49.5	25	.0066
6	20.8	80	66	54.5	48.5	25	.0075
7	20.9	79	66	55	48	25	.0070
TEST CONDITION D							
1	--	61	45	36.5	27	50	.0054
2	--	64.5	46	36	27.5	50	.0058
3	--	65.5	47	37.5	28.5	50	.0056
4	--	66	48	38.5	29.5	50	.0054
5	18.2	68.5	56	47	40.5	25	.0073
6	18.2	59	50	42	37	25	.0066
7	18.2	60.5	50	42.5	37.5	25	.0071



TEST CONDITION A - Without Water
Without Insulation
Paper Speed - 25 mm/sec
0
2
3
4
5
Time (seconds)
CHART NO. RA-2911-30
BRUSH
TEST CONDITION B - With Water
Without Insulation
Paper Speed - 25 mm/sec
0
2
3
4
5
Time (seconds)
PRINTED IN U.S.A.
CHART NO RA-2911-30
BRUSH INSTRUMENTS
TEST CONDITION C - Without Water
With Insulation
0
2
3
4
5
(seconds)
CHART NO. RA-2911-30
BRUSH INSTRUMENTS
TEST CONDITION D - With Water
With Insulation
Paper Speed - 25 mm/sec
0
2
3
4
5
INSTRUMENTS
DIVISION OF CLEVITE CORPORATION
CLEVELAND, OHIO
PRINTED IN U.S.A

FIGURE 9



Test Condition A - Without Water
Without Insulation
Paper Speed - 125 mm/sec
0
1
2
Time (seconds)
CHART NO. RA-2911-30
BRUSH INSTRUMENTS
DIVISION OF CLEVITE CORPORATION
Test Condition B - With Water
Without Insulation
Paper Speed - 125 mm/sec
BRUSH INSTRUMENTS
DIVISION OF CLEVITE CORPORATION
CLEVELAND OHIO
PRINTED IN U.S.A.
Test Condition C - Without Water
With Insulation
Paper Speed - 125 mm/sec
Time (seconds)
CHART NO. RA-2911-30
BRUSH INSTRUMENTS
DIVISION OF CLEVITE CORPORATION
CLEVELAND, OHIO
PRINTED IN U.S.A
Test Condition D - With Water
With Insulation
Paper Speed - 125 mm/sec
Time (seconds)
CHART NO. RA-2911-30
BRUSH INSTRUMENTS
DIVISION OF CLEVITE CORPORATION
CLEVELAND, OHIO
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FIGURE 10